

Please get out:

- * The Ch. 7 notes from Friday
- * Your HW worksheet

On the blank piece of paper, please write

1) Your name at the top, and Ch. 7 & 8

2) Title the paper: **PROBABILITY RULES CHEAT SHEET**

Example:

John Smith

Ch. 7 & 8

PROBABILITY RULES CHEAT SHEET

Introductory Vocab (use the book, chapter 7, to get the definitions)

Random (trials)- * Individual outcomes are uncertain

* We see a pattern in many trials (in the long run)

Ex: flip coin, simulation Tony Gwynn

Probability- * The chance (likelihood) of something happening

Experimental Probability- the probability that ACTUALLY happened in the experiment.

Ex: If I toss a coin 30 times, and get 12 heads, what's the experimental prob. of getting heads?

$$P(H) = 12/30$$

Theoretical Probability- The probability that SHOULD have happened in the experiment.

Ex: Using the same coin tossing situation above, what's the theoretical prob. of getting heads?

$$P(H) = 0.50 = 50\% = \frac{1}{2}$$

Probability Models- A list of all the possible outcomes and the probability of each outcome.

x	0	1	2	3
$P(x)$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{8}$...

Sample Space- All the possible outcomes (List)

Example: What's the sample space for the Tony Gwyn experiment? Hit or Not hit (miss)

How about for the spinner experiment? Red, Blue, Yellow, Green

What about the rolling 2 dice experiment? 2, 3, 4, 5, , 9, 10, 11, 12

Event- An outcome, or a set of outcomes

Ex: When rolling one dice, outcomes could be: Evens a 5, etc.

****READ PAGE 318 – 319: “The Myths”**

Probability Notation:

- $A, B, C, \text{etc.}$ = events that can happen
- $P(A)$ = the probability of event A happening
- S = the sample space

$P(H)$
 $P(\text{Jack})$

$P($

CHAPTER 7: PROBABILITY

General Set Theory

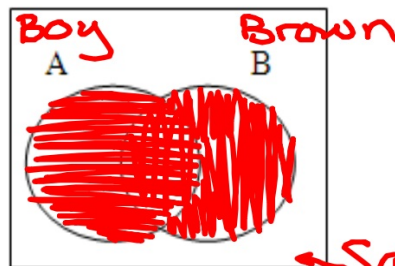
Union:

- Meaning:
- Symbol:
- Example 1:

addition (OR)

\cup

$A \cup B$



← Sample Space

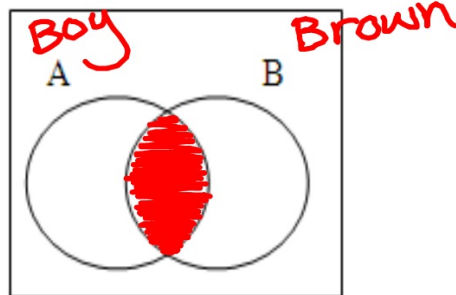
- Example 2: $\text{Set A} = \{2, 4, 6, 8, 10, 12\}$
 $\text{Set B} = \{1, 2, 3, 4, 5, 6, 7\}$

$A \cup B = A \text{ or } B = \{$

$\{1, 2, 3, 4, 5, 6, 7, 8, 10, 12\}$

Intersection:

- Meaning: **Overlap, AND**
- Symbol: **\cap** **$A \cap B$**
- Example 1:

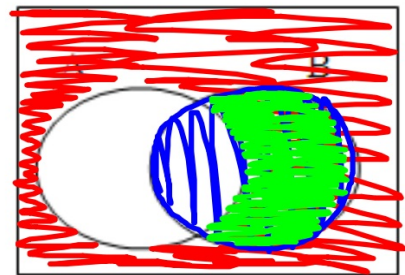
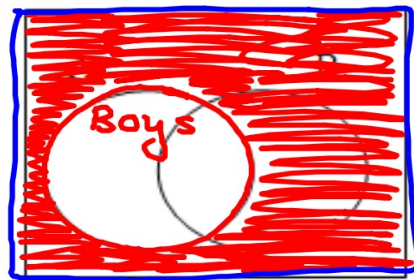


- Example 2: Set A = {2, 4, 6, 8, 10, 12}
Set B = {1, 2, 3, 4, 5, 6, 7}

$$A \cap B = A \text{ and } B = \{2, 4, 6\}$$

Complement:

- Meaning: "NOT"
An event not happening Brown^c
- Symbol: A^c A' Boy^c = not a boy
- Example 1: Shade A^c Shade $A^c \cap B$ ← overlap



- Example 2: Set $A = \{2, 4, 6, 8, 10, 12\}$
 $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$ = sample space

$$A^c = \{1, 3, 5, 7, 9, 11, 13, 14, 15\}$$

TRY THESE:

Sample Space = {1, 2, 3, 4, ..., 22, 23, 24, 25}

A = {1, 3, 6, 7, 8, 10, 11, 13, 14, 15}

B = {3, 5, 7, 9, 11, 13, 15, 17, 19, 21}

C = {2, 4, 6, 8, 10, 12, 14, 16, 20, 22, 24}

1) What is $A \cap B$?

$\{3, 7, 11, 13, 15\}$

2) What is $A \cup B$?

$\{1, 3, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15, 17, 19, 21\}$

3) What is B^c ?

$\{1, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 23, 24, 25\}$

4) What is $C \cap B^c$?

$\{2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24\}$

9) What is $P(A \cap B)$?

$\frac{5}{25} = \frac{1}{5}$

10) What is $P(A \cup B)$?

$\frac{15}{25}$

11) What is $P(B^c)$?

$\frac{15}{25}$

12) What is $P(C \cap B^c)$?

$\frac{12}{25}$

5) What is $A \cap B \cap C$? *empty set*
 $\{ \emptyset \}$

13) What is $P(A \cap B \cap C)$?
 $0/25 = 0\%$

6) What is $A \cup B \cup C$?

14) What is $P(A \cup B \cup C)$?

$\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, \dots, 17, 19, \dots, 22, 24\}$
all except 18 and 23 and 25

$22/25$

7) What is $A \cap C$?

15) What is $P(A)$?

$\{6, 8, 10, 14\}$

$10/25$

8) What is C^c ?

16) What is $P(B)$?

$\{1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25\}$ $10/25$

Try these.... SET THEORY

$S = \text{sample space} = \{1, 3, 4, 5, 6, 7, 8, 11, 13, 15, 16, 17, 19, 21, 22, 23, 26, 28, 29, 30\}$

$E = \{1, 3, 5, 7, 15, 17, 23\}$

$H = \{4, 6, 8, 16, 22, 26, 28, 30\}$

$D = \{3, 4, 5, 6, 7, 8, 11, 13, 15\}$

$C = \{15, 16, 17, 19, 21, 23, 26\}$

$A = \{1, 5, 15, 16, 22, 26, 28, 30\}$

$B = \{7, 13, 15, 16, 17, 22, 28, 30\}$

HW

Assuming all data in S are equally likely, find each of the following:

a. $P(H) =$

b. $P(E) =$

c. $E \cap H = \{$

d. $P(E \cap H) =$

e. $D \cup B = \{$

g. $P(D) =$

i. $P(A^c) =$

k. $P(B) =$

m. $P(D^c) =$

o. $P(E \cap C) =$

q. $P(E \cup H) =$

f. $P(D \cup B) =$

h. $P(A) =$

j. $P(C^c) =$

l. $P(E \cap B) =$

n. $P(A \cap B) =$

p. $P(A \cup B) =$

r. $P(C \cap A) =$

← HW

Probability Rules

- Let A and B be events
- Let S = sample space
- Let A^c = the complement of event A

• List the 3 probability rules:

(1) $0 \leq P(A) \leq 100\%$

(2) $P(S) = 100\%$

(3) $P(A^c) = 100\% - P(A)$

Write these 3 rules on your probability rules cheat sheet!!

Example 1: If the probability of hitting a homerun is 30%, what's the probability of not hitting a homerun?

* $P(H) = 30\% = 0.30$

$P(H^c) = 1 - 0.30 = 0.70$ or 70%

Example 2: If there are only 8 different blood types, fill in the chart below:

1-

Prob. Model

Type	A+	A-	B+	B-	AB+	AB-	O+	O-
Probability	0.16	0.14	0.19	0.17	?	0.07	0.1	0.11

0.06

Example 3: Las Vegas Zeke, when asked to predict the ACC basketball Champion, follows the modern practice of giving probabilistic predictions. He says, "UNC's probability of winning is twice Duke's. NC State and UVA each have probability 0.1 of winning, but Duke's probability is three times that. Nobody else has a chance." Has Zeke given a legitimate assignment of probabilities to all the teams in the conference? Why or why not?

$P(\text{UNC}) = 0.6$

$P(\text{NCState}) = 0.1$

$1.1 = P(S)$

$P(\text{Duke}) = 0.3$

$P(\text{UVA}) = 0.1$

Going back to the examples from before...

Ex #1: There are only 8 different blood types, given in the chart below:

Type	A+	A-	B+	B-	AB+	AB-	O+	O-
Probability	0.16	0.14	0.19	0.17	0.06	0.07	0.1	0.11

What is the probability of being either Type A+ or B-?

$$P(A+ \cup B-) = 0.16 + 0.17 = 0.33$$

What is the probability of being either Type O- or O+?

$$P(O- \cup O+) = P(O) = 0.1 + 0.11 = 0.21$$

What is the probability of being either Type AB+ or A+?

$$P(AB+ \cup A+) = 0.06 + 0.16 = 0.22$$

So... to find the probability of event A OR event B we...

$$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B)$$

$$P(K) =$$

$$P(L) =$$

Ex #2:

We are picking one card out of a standard 52-card deck (no jokers).

What is the probability of picking a diamond? $P(D) = \frac{13}{52}$

What is the probability of picking a 3? $P(3) = \frac{4}{52}$

What is the probability of picking a diamond or a 3?

$$P(D \cup 3) = P(D) + P(3) = \frac{13}{52} + \frac{4}{52}$$

What is the probability of picking a black card?

$$P(B) = \frac{26}{52}$$

What is the probability of picking a Jack?

$$P(J) = \frac{4}{52}$$

What is the probability of picking a black card or a Jack?

$$P(B \cup J) = \frac{26}{52} + \frac{4}{52} - \frac{2}{52} = \frac{28}{52}$$

So... to find the probability of event A OR event B we....

$$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

But why didn't we do this in EX #1? What was different about EX #1?

no overlap

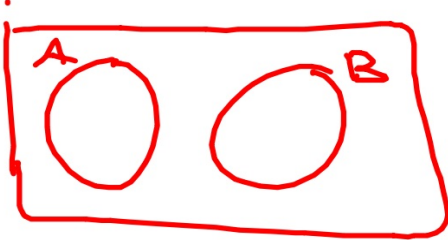
This is called...

and it means that...

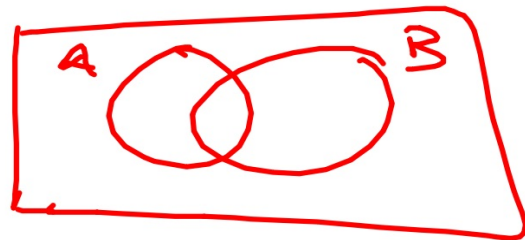
disjoint = no intersection

Visuals: Venn Diagrams

DISJOINT



NOT DISJOINT:



RULE for Unions (OR):

Write these rules down on your probability rules cheat sheet:

Unions:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = 0$$

SPECIAL CASE: **Disjoint (A and B do not overlap)
no intersection

$$P(A \cup B) = P(A) + P(B)$$

Try this:

1) $P(K) = 0.34$ and $P(B) = 0.22$ and $P(K \cap B) = 0.13$.

a. Are K and B disjoint? Why or why not?

No. $P(K \cap B) \neq 0$.

b. What is $P(\overset{K \cup B}{\cancel{K \cap B}})$?

$$\begin{aligned} P(K \cup B) &= P(K) + P(B) - P(K \cap B) \\ &= 0.34 + 0.22 - 0.13 = \textcircled{0.43} \end{aligned}$$

2) $P(H) = 0.57$ and $P(J) = 0.32$ and $P(J \cap H) = 0.20$.

a. Are $\overset{H}{\cancel{H}}$ and $\overset{J}{\cancel{J}}$ disjoint? Why or why not?

No. $P(H \cap J) \neq 0$.

b. What is $P(\overset{H \cup J}{\cancel{H \cap J}})$?

$$\begin{aligned} P(H) + P(J) - P(H \cap J) \\ 0.57 + 0.32 - 0.20 = \textcircled{0.69} \end{aligned}$$

NEW EXAMPLES!

EX #3: The probability of hitting a homerun is 30%. What is the probability of hitting 2 HRs in a row?

$$P(H) = 0.30 \quad P(H \cap H) = (0.3)(0.3)$$

How about 3 HRs in a row?

$$P(H \cap H \cap H) = 0.3 \cdot 0.3 \cdot 0.3 = 0.027$$

How about 4 HRs in a row?

How about 2 HRs then a non-homerun?

$$P(H \cap H \cap H^c) = (0.3)(0.3)(0.7)$$
$$\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = 0.063$$

EX #4: Picking poker chips from a bag with replacement:

I have a bag that has 10 poker chips in it. 3 are red, 2 are green, and 5 are blue.

$$P(R) = 3/10 \quad P(G) = 2/10 \quad P(B) = 5/10$$

What is the probability of picking a red chip and a blue chip?

$$P(R \cap B) = (3/10)(5/10) = 0.15$$

What is the probability of picking a green chip and a red chip?

$$P(G \cap R) = (2/10)(3/10) = 0.06$$

What's the probability of picking a blue chip and a blue chip?

$$P(B \cap B) = (5/10)(5/10) = 0.25$$

So... to find the probability of event A AND event B we....

$$P(A \text{ and } B) = P(A \cap B) = P(A) \cdot P(B)$$

EX #5: Picking poker chips from a bag *without* replacement

I have a bag that has 10 poker chips in it. 3 are red, 2 are green, and 5 are blue.

What is the probability of picking a red and then a blue?

What is the probability of picking a blue and then a blue?

What is the probability of picking a green and then a red?

VOCAB: Conditional Probabilities:

So... to find the probability of event A AND event B we....

$$P(\text{A and B}) = P(A \cap B) =$$

But why didn't we do this in EX #3 and 4? What was different about EX #5?

This is called... and it means that...

RULE for Intersections (AND):

Write this new rule down on your probability rules cheat sheet:

INTERSECTIONS:

$$P(A \cap B) = P(A) * P(B | A) \quad \text{or} \quad P(B | A) = \frac{P(A \cap B)}{P(A)}$$

****SPECIAL CASE: INDEPENDENT** (A happening doesn't affect B happening)

$$P(A \cap B) = P(A) * P(B)$$

Try these:

- 1) $P(Y) = 0.35$ and $P(K) = 0.41$ and $P(Y \cap K) = 0.13$.
 - a. Are Y and K independent? Why or why not?
 - b. What is $P(Y|K)$?
 - c. What is $P(K|Y)$?

- 2) $P(H) = 0.55$ and $P(D) = 0.37$ and $P(D|H) = 0.20$.
 - a. Are H and D independent? Why or why not?
 - b. What is $P(D \cap H)$?
 - c. What is $P(H|D)$?

Try the probability rules worksheet #1

